Fast Linear Algebra-Based Triangle Analytics with Kokkos Kernels

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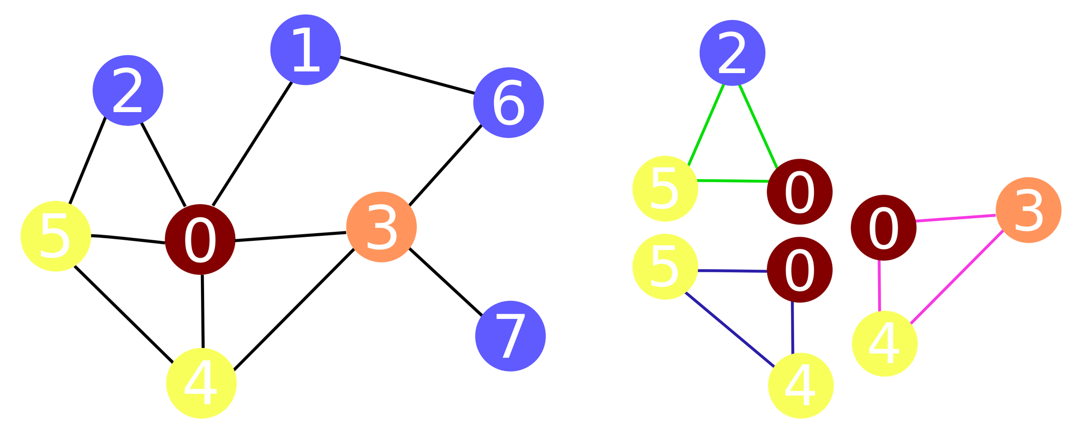
Introduction



A Toy Graph   Triangles

Triangle-based analytics are important in many larger data-analytics based applications. Previously, a highly efficient linear algebra-based algorithm has been developped in Kokkos-Kernels

Primary contributions:

- We improve upon that work by developing an SpGEMM implementation that relies on a highly efficient, work-stealing, multithreaded runtime.
- We demonstrate that our implementation results in improving the runtime up to $5\times$ to $12\times$ on different architectures

Triangle Counting

A triangle can be defined as a set of three mutually adjacent vertices in a graph.

Triangle Counting Problem

Given a graphs $G = \{V, E\}$, the triangle counting problem is to find the number (T) of all set of three vertices, $u, v, w \in V$, such that:

$$T = \mid \{u, v, w \mid (u, v), (v, w), (w, u) \in E\} \mid .$$

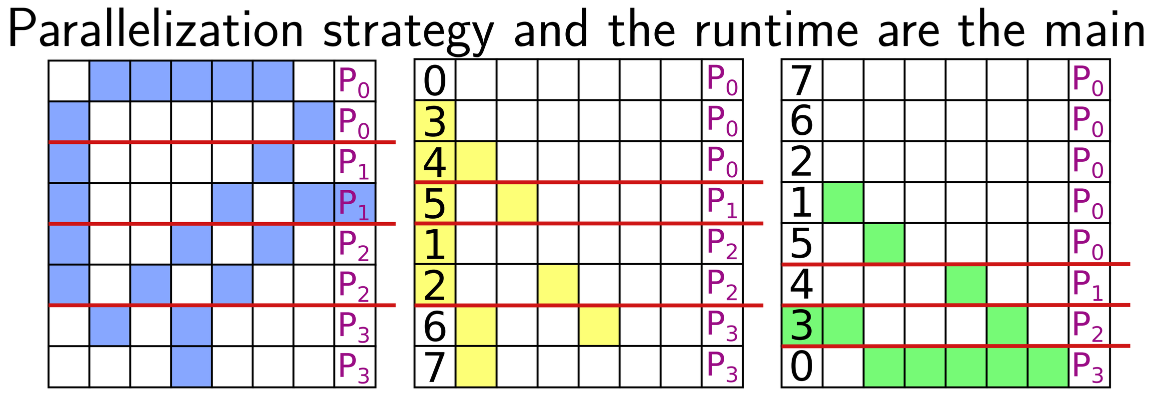
Linear Algebra Formulations

Two linear-algebra based formulations of triangle counting that are based on the adjacency matrix of the graph: L and U represent lower and upper parts

- LU algorithm; $D = (L \cdot U) .* L$
  - (Pro): Low operation count,
  - (Con): Poor scalability
- LL algorithm; $D = (L \cdot L) .* L$
  - (Pro): Good scalability,
  - (Con): More operations than LU

kkTri-Cilk Algorithm

Parallelization strategy and the runtime are the main differences between KKTri-Cilk and KKTri.

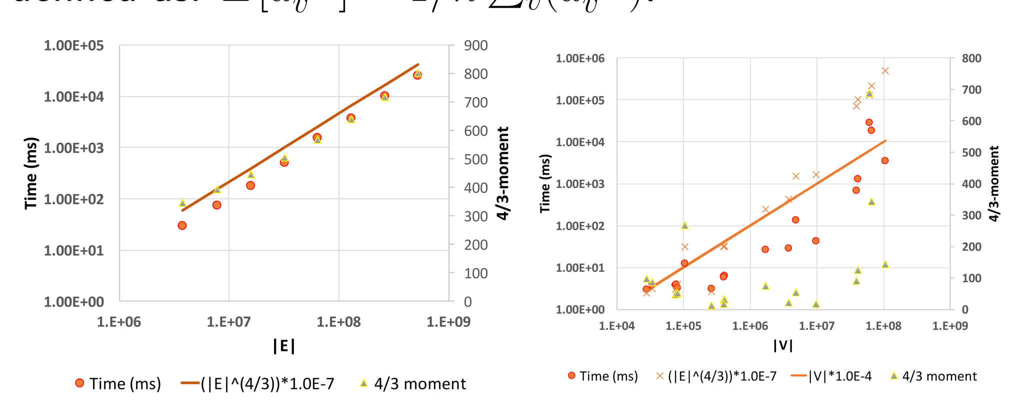


Matrix Form   LL Alg: L Matrix   LU Alg: L Matrix

- Rows are ordered for avoiding computation:
  - LL; in decreasing degree,
  - LU; in increasing degree
- Balance number of non-zeros within each partition.
- Each partition is spawned (in parallel) as a task.
  - A task runs matrix matrix multiplication within a partition.

Triangle Counting Scalability

Letting d_v be the degree of vertex v, the $4/3$-moment is defined as: $E[d_v^{4/3}] = 1/n \sum_v (d_v^{4/3})$.



Scale Graphs   Non Scale Graphs

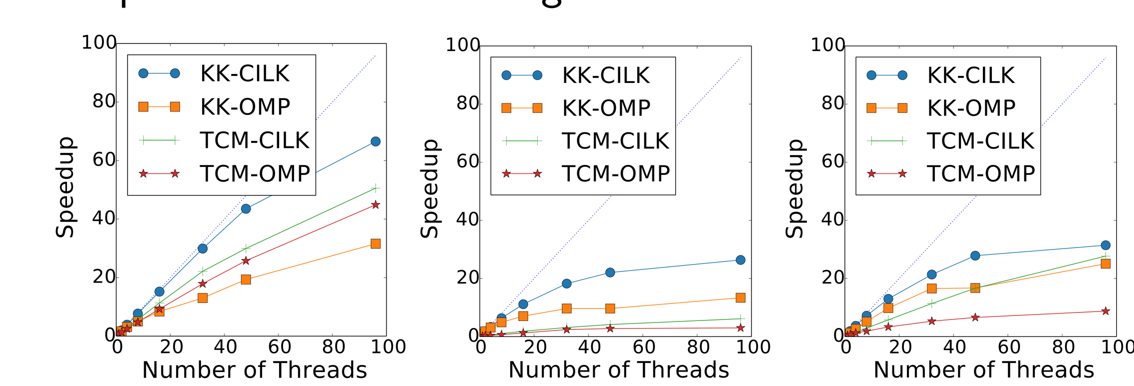
- $m^{4/3}$ (Graph Challenge 2017 regression line) predicts the trends effectively in the Scale graphs.
- $4/3$-moment is correlated with the runtime for both Scale and Non-Scale graphs.

Graph Type	Time-per-vertex		Time-per-edge	
	$E^{\frac{4}{3}}$	$\frac{4}{3}$-moment	$E^{\frac{4}{3}}$	$\frac{4}{3}$-moment
Scale graphs	0.90	0.98	0.89	0.98
Non-scale graphs	0.26	0.95	0.02	0.76

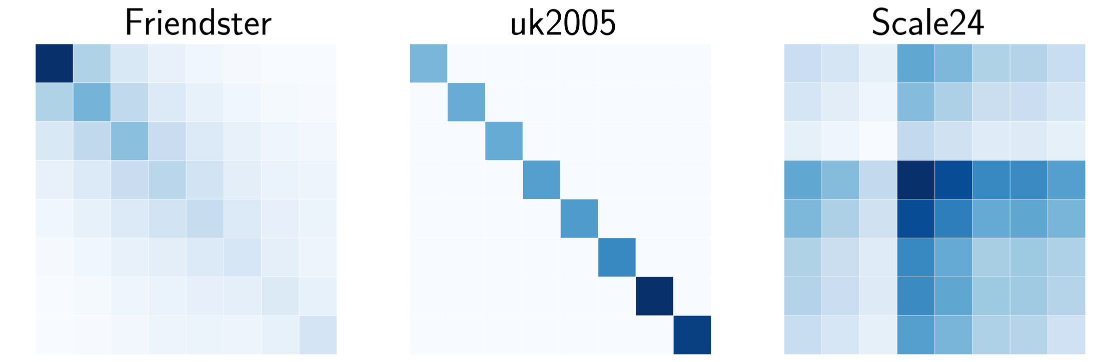
Per-vertex/edge runtimes are highly correlated with the 4/3-moment, as predicted "by past theory".

Experiments: Strong Scaling

Following figures show strong scaling experiments for OpenMP and Cilk implementations of two algorithms.



Friendster   uk2005   Scale24

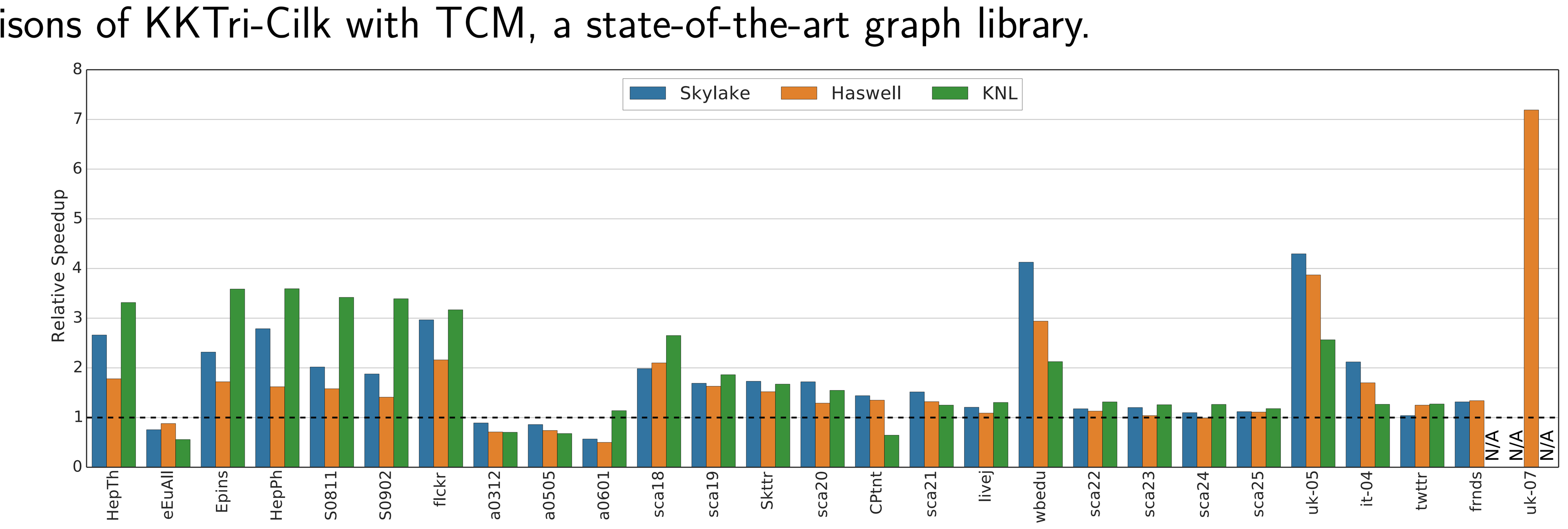


Friendster   uk2005   Scale24

- KKTri-Cilk scales the best in all three problems
- uk-2005 achieves best rate: highly local computations.
- scale24 achieves worst rate: Poor cache usage.
- Friendster graph's distribution is in between (best scalability).

Experiments: Relative Speedup

Comparisons of KKTri-Cilk with TCM, a state-of-the-art graph library.



- KKTri achieves up to $7\times$ speedup on graphs that have a good natural ordering such as wb-edu, uk-2005, and uk-2007. KKTri outperforms TCM in 23 of 27 cases.

Experiments: Dataset and Peak Rate

Times highlighted in green when KKTri-Cilk is the fastest.

- 10^9 barrier is passed for the uk-2005 matrix and wb-edu graph.
- A high correlation (0.91) between the conductance and the rate.

Data Set	$1 - C^d$	Time (s)	Rates		
			Skylake	Haswell	KNL
cit-HepTh	0.141	0.003	1.20E+08	8.24E+07	1.54E+07
email-EuAll	0.112	0.003	1.16E+08	1.10E+08	2.16E+07
soc-Epinions1	0.086	0.004	1.06E+08	6.72E+07	2.44E+07
cit-HepPh	0.091	0.004	1.11E+08	8.77E+07	2.47E+07
soc-Slashdot0811	0.067	0.004	1.18E+08	7.97E+07	2.71E+07
soc-Slashdot0902	0.069	0.003	1.57E+08	8.64E+07	2.77E+07
flickrEdges	0.098	0.013	1.85E+08	1.15E+08	2.99E+07
amazon0312	0.229	0.006	3.87E+08	2.51E+08	9.34E+07
amazon0505	0.233	0.006	3.79E+08	2.75E+08	9.36E+07
amazon0601	0.276	0.006	4.17E+08	2.87E+08	9.81E+07
scale18	0.059	0.031	1.24E+08	1.07E+08	2.88E+07
scale19	0.058	0.075	1.04E+08	8.06E+07	2.79E+07
as-Skitter	0.17	0.026	4.23E+08	3.23E+08	1.23E+08
scale20	0.059	0.184	8.53E+07	5.63E+07	2.50E+07
cit-Patents	0.027	0.028	5.82E+08	4.21E+08	1.22E+08
scale21	0.059	0.511	6.21E+07	4.78E+07	2.01E+07
soc-LiveJournal1	0.242	0.137	3.14E+08	2.28E+08	1.07E+08
wb-edu	0.938	0.042	1.10E+09	6.55E+08	1.48E+08
scale22	0.058	1.581	4.05E+07	3.50E+07	1.71E+07
scale23	0.059	3.786	3.41E+07	2.62E+07	1.45E+07
scale24	0.059	10.282	2.53E+07	2.04E+07	1.21E+07
scale25	0.059	25.652	2.04E+07	1.88E+07	9.11E+06
uk-2005	0.925	0.684	1.14E+09	9.35E+08	2.59E+08
it-2004	0.942	1.293	7.95E+08	5.86E+08	1.47E+08
twitter	0.126	28.359	4.24E+07	4.46E+07	N/A
friendster	0.182	18.552	9.74E+07	7.93E+07	N/A
uk-2007	0.968	3.545	9.31E+08	7.49E+08	N/A

Conclusion

- KKTri-Cilk surpasses 10^9 for the rate measure.
- KKTri-Cilk is faster on 63 of 78 instances
- KKTri-Cilk is faster than state-of-the-art graph based implementation (up to $7\times$)
- We corroborate that the scalability of the triangle counting is bounded by $O(n)$ when the $4/3$-moment is bounded
- We show correlation between the high rates achieved and the conductance of the graph

References

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